



## Normative framework of laser beam measurement

For the description of laser beams there are historically many different approaches and definitions. However, if data is to be exchanged between different actors, it is essential to agree on uniform definitions. In order to create a standardization for the characteristics of laser radiation, national and international standards have been defined. The standardization by DIN and ISO thus forms the formal framework for the reproducible measurement of laser beams according to internationally recognized rules.

This frame sets standards in the interpretation of measurement results. PRIMES, as a leading and independent manufacturer of measuring equipment, is significantly involved in the definition of the standards. The relevant DIN standards for the measurement and characterization of laser beams are: 11145, 11146, 11554, 11670 and 13694.

These standards are the basis for measurements with highest accuracy. All PRIMES measurements are based on these standards. The most important formulas for the interpretation of the measurement results and characteristics are summarized below.

## Central formulae for beam measurement

### Beam parameter product BPP

The beam parameter product is a physical parameter describing the beam quality, and hence the focusability, of a laser beam.

$$\frac{\Theta \cdot d_0}{4} = \frac{\Theta \cdot r_0}{2} = \frac{\lambda \cdot M^2}{\pi}$$

$\Theta$  = full beam divergence angle

$d_0$  = beam waist diameter

$r_0$  = beam waist radius

$M^2$  = beam quality factor

$\lambda$  = wavelength

### Beam quality factor $M^2$

The beam quality factor  $M^2$  is a dimensionless quantity that characterizes a laser beam: the larger  $M^2$ , the more difficult it is to focus the beam; i.e. the larger is the smallest possible focus diameter.

$$M^2 = \frac{1}{k}$$
$$k = \frac{4 \cdot \lambda}{\pi} \cdot \frac{F}{d_0}$$
$$r_0 = \frac{2 \cdot \lambda}{\pi} \cdot \frac{f}{d_s} \cdot M^2$$

$\lambda$  = wavelength

$d_0$  = beam waist diameter (also see **beam parameter product BPP**)

$k$  =  $1/M^2$  = beam propagation ratio

$f$  = focal length of focusing optics

$d_s$  = raw beam diameter = diameter of collimated beam before focusing optics

$F$  = F-number =  $f/d_s$

A Gaussian beam has  $M^2 = 1$ , a real single-mode beam has an  $M^2$  between 1 and 1.2. Multi-mode beams typically have  $M^2 > 5$ .  $M^2$  is a universal parameter of beam quality that can be used to compare different beam sources.

**Rayleigh length  $z_R$  (colloquial: depth of focus)**

The Rayleigh length is the distance along the propagation direction of a beam from the waist to the place where the area of the cross section is doubled.

$$z_R = \frac{\pi \cdot r_0^2}{\lambda \cdot M^2}$$

$z_R$  = Rayleigh length

$\lambda$  = wavelenght

$r_0$  = beam waist radius

**Formulas for pulse applications**

$$P_p = \frac{E_p}{t_p}$$

$$\bar{p} = E_p \cdot f_p$$

$$I = \frac{P_p}{A}$$

$P_p$  = pulse power

$E_p$  = pulse energy

$t_p$  = pulse duration

$\bar{p}$  = average power

$f_p$  = pulse frequency

$I$  = power density

$A$  = beam surface

**Formulas for fiber applications**

$$NA = n \cdot \sin\left(\frac{\alpha}{2}\right)$$

NA = numerical aperture

$n$  = refractive index before fiber

$\alpha$  = full opening angle of the fiber

**Ellipticity, aspect ratio  $\varepsilon$** 

Parameter for the quantification of the circularity or square shape of a power/energy density distribution.

$$\varepsilon = \frac{\min\{d_x, d_y\}}{\max\{d_x, d_y\}}$$

$$0 < \varepsilon \leq 1$$

$d_{x,y}$  = beam diameter along the principal axes of an elliptic beam

For an elliptical beam,  $\varepsilon$  corresponds to the ratio of the principal axes and is called ellipticity. For rectangular beams,  $\varepsilon$  corresponds to the ratio of the side edges and is called aspect ratio.  $\varepsilon = 1$  corresponds to a round or square beam.



## Formulas for top-hat beams

### Edge steepness $s_{\eta,\epsilon}$

Normalized difference of the confined irradiation areas  $A_\eta$  and  $A_\rho$ .

$$s_{\eta,\rho} = \frac{A_\eta - A_\rho}{A_\eta}$$

$$0 \leq \rho < \eta \leq 1$$

$$0 < s_{\eta,\epsilon} < 1$$

$A_{\eta,\rho}$  = geometric irradiation area at clip level  $\eta, \rho$

$\rho, \eta$  = clip level. Maximum intensity is set on 1, 0 corresponds to the offset of the measurement. In PRIMES measurements applies:  $\rho = 0.1$  and  $\eta = 0.9$ .

The steeper the edge, the smaller  $s$  becomes. A vertical edge (ideal top-hat beam) has the value  $s = 0$ . A Gaussian beam has the value  $s = 0.96$ .

### Flatness $F_\eta$

Ratio of the confined average power density (energy density) to the maximum power/energy density of the distribution. The flatness describes the height of the highest outliers above the average plateau of the power density.

$$F_\eta = \frac{E_{\eta\text{ave}}}{E_{\text{max}}}$$

$$0 < F_\eta \leq 1$$

$\eta$  = clip level 0.9 (see **edge steepness**)

$E_{\eta\text{ave}}$  = average power density from clip level  $\eta$

$E_{\text{max}}$  = maximum power density

For a perfectly homogeneous profile,  $F = 1$ . The higher the outliers, the smaller  $F$  becomes.

### Uniformity $U_\eta$

Root mean square (rms) of the power-/energy distribution from its confined mean value.

$$U_\eta = \frac{1}{E_{\eta\text{ave}}} \frac{1}{A_\eta} \sqrt{\iint [E(x, y) - E_{\eta\text{ave}}]^2 dx dy}$$

$\eta$  = clip level 0.9 (see **edge steepness**)

$E$  = power density at  $x, y$

$A_\eta$  = geometric irradiation area (see **edge steepness**)

$E_{\eta\text{ave}}$  = average power density from clip level  $\eta$

$U$  is given in % referred to  $E_{\eta\text{ave}}$ . A perfectly homogeneous profile has  $U = 0$  %.

**Tilt angle  $N_{xy}$** 

Tilt of the power density in the region of maximum beam intensity, normalized to the mean power density.

$$E(x, y) = a \cdot x + b \cdot y + c$$

$$N_x = \frac{a \cdot d_x}{E_{ave}}$$

$$N_y = \frac{b \cdot d_y}{E_{ave}}$$

$E$  = power density at  $x, y$

$d_{x,y}$  = beam diameter in directions  $x$  and  $y$

$E_{ave}$  =  $E_{0.9ave}$  average power density (see **flatness**)

$N$  indicates the tilt of a plane describing the distribution of the power density above a clip level of 0.9. The numerical value of  $N$  corresponds to the absolute difference between the extreme values of the power density, in % with respect to the mean power density  $E_{ave}$ . A slope of 2 % means that the crown of the top-hat distribution changes from one side of the beam to the other by 2 % of the mean beam intensity  $E_{ave}$ .



## Formulas for analysis

### Center of gravity $\langle x \rangle, \langle y \rangle$

First moment of the surface-intensity-distribution to determine the center of gravity.

$$\langle x \rangle = \frac{\iint x \cdot E(x, y) \, dx dy}{\iint E(x, y) \, dx dy}$$

$$\langle y \rangle = \frac{\iint y \cdot E(x, y) \, dx dy}{\iint E(x, y) \, dx dy}$$

$E$  = Power density at  $x, y$

### Radius from moments of second order $r_x, r_y$

Second central moment of surface-intensity-distribution to determine variance and beam radius.

$$\langle x^2 \rangle = \frac{\iint (x - \langle x \rangle)^2 \cdot E(x, y) \, dx dy}{\iint E(x, y) \, dx dy}$$

$$\langle y^2 \rangle = \frac{\iint (y - \langle y \rangle)^2 \cdot E(x, y) \, dx dy}{\iint E(x, y) \, dx dy}$$

$$r_x = 2 \cdot \sqrt{\langle x^2 \rangle}$$

$$r_y = 2 \cdot \sqrt{\langle y^2 \rangle}$$

$$r = 2 \cdot \sqrt{\frac{1}{2} \cdot (\langle x^2 \rangle + \langle y^2 \rangle)}$$

$E$  = power density at  $x, y$

$\langle x \rangle, \langle y \rangle$  = beam center of mass

$\langle x^2 \rangle, \langle y^2 \rangle$  = Second central moment, variance